

# Technical Comments

## Comments on "Buckling of Cylindrical Shells with Eccentric Spiral-Type Stiffeners"

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AS assumed by author Soong in Ref. 1, there is indeed a typographical error in the reported dimension of the rib depth for specimen #4, Pt. 2 of Ref. 2. The  $d$  dimension reported there as 0.293 in. should have been denoted as 0.193 in. The calculated buckling value of Ref. 2 for this case, 739 lb/in., was based upon the correct dimension of 0.193 in.

In Table 1 of Mr. Soong's article, it is not clear that column 3, shown there as "Ref. 6,  $\theta = 45^\circ$ ,  $\bar{N}_x$ ," refers to calculated values for buckling as developed in that reference. Comparison of the theoretical values for the two theories shows (with the exception of #4, Pt. 2) negligible difference between them. Additional bending theory developed at McDonnell Douglas has also confirmed the postulates of local stability as assumed in Ref. 2, demonstrating that there is no significant difference between theoretical values for axial compression and bending buckling loads per inch for the case of  $45^\circ$  eccentric stiffened cylinders. Thus, the simpler theory developed in Ref. 2 is adequate for computing buckling values for almost all cases of practical interest for  $45^\circ$  eccentric stiffened cylinders.

It should be emphasized that weight comparisons for optimum design should be based upon all modes of buckling failure for the construction, viz., general instability, between ring buckling, if applicable, skin buckling, and rib crippling; otherwise erroneous conclusions may be reached for optimum configurations.

### References

<sup>1</sup> Soong, T. C., "Buckling of Cylindrical Shells with Eccentric Spiral-Type Stiffeners," *AIAA Journal*, Vol. 7, No. 1, Jan. 1969, pp. 65-72.

<sup>2</sup> Meyer, R. R., "Buckling of  $45^\circ$  Eccentric-Stiffened Waffle Cylinders," *Journal of the Royal Aeronautical Society*, Vol. 71, July 1967, pp. 516-520.

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## Reply by Author to R. R. Meyer

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THE purpose of the author's paper<sup>1</sup> was the derivations and parametric studies of a general method for instability analysis of cylinders with arbitrary skew stiffeners, under various loading conditions. It is possible and might be expected that for a specific type of load, such as a uniform axial

compression, and a fixed geometry, such as the  $45^\circ$  skew angle, a simpler method could yield comparable results, as has been shown in Ref. 2. But, this is not the point. Also, the possibility that the spiral-type stiffening might have weight advantages over the conventional types was suggested under the assumptions of general instability. Skin and panel types of instability are understood as different issues.

In R. R. Meyer's Comment, an interesting point has been brought up, namely, the applicability of the local instability theory that Meyer<sup>2</sup> has utilized to predict nonuniform axial loading cases by means of uniform axial compression analyses. It is a good opportunity to discuss this theory briefly.

There is no clear, definitive description of the local instability theory in literature. From the derivations of Vlasov<sup>3</sup> and of Mushtari and Galimov,<sup>4</sup> one might tentatively give the following description. The local instability theory asserts that instability of a local nature is possible and may provide as a lower limit for the general instability of the shell if the membrane stresses in the local region are large enough to cause a loss of stability in that locality.

In the derivations, the original general differential equations of equilibrium which are linear<sup>3</sup> or nonlinear<sup>4</sup> with variable coefficients had been reduced to a pair of linear equations with constant coefficients of the Donnell form<sup>3</sup> by simply assigning the principal curvatures at the local points as constant and neglect complications from the boundary. It seems clear that with these generous assumptions and omission of boundary constraints, the local instability theory is not intended as an exact theory, presumably as an estimate of lower bound. The quality of such estimates can be found only after performing an appropriate analysis, say, a bending analysis for a bending problem.

It is interesting to review briefly the related literature. The authors of Refs. 5-9 performed analyses on cylinders, sandwich types or isotropic, under variable axial compressive loads and found that predictions based on uniform axial load would have obtained the same buckling stress. On the other hand, linear<sup>10,11</sup> and nonlinear<sup>12,13</sup> analyses performed on isotropic cylinders<sup>10,12,13</sup> and stiffened cylinders<sup>11</sup> under bending load showed that bending stress is 1.3 times or more greater than the compressive buckling stress. In addition, experiments performed on cylindrical isotropic shells,<sup>14,18</sup> sandwich shells,<sup>15</sup> and isotropic elliptic shells,<sup>16,17</sup> and for materials ranging from aluminum alloy<sup>15-17</sup> to steel and brass,<sup>14</sup> found that for the same cylinder bending-buckling stress is 25 to 60% greater than the corresponding compression-buckling stress, the exact value depending on the  $R/t$  ratio.

In view of these analyses and tests reported by previous investigators, one might comment that the local instability theory should be used with caution, and preferably only as a lower bound. It is doubtful that it will remain as a lower bound when stiffening and anisotropy are added to shell problems.

Finally, the author would like to thank R. R. Meyer for furnishing the information in his Comment regarding a typographical error in his reported dimension of specimen #4, Pt. 2.

### References

<sup>1</sup> Soong, T. C., "Buckling of Cylindrical Shells with Eccentric Spiral-Type Stiffeners," *AIAA Journal*, Vol. 7, No. 1, Jan. 1969, pp. 65-72.

<sup>2</sup> Meyer, R. R., "Buckling of  $45^\circ$  Eccentric-Stiffened Waffle Cylinders," *Journal of the Royal Aeronautical Society*, Vol. 71, July 1967, pp. 516-520.

<sup>3</sup> Vlasov, V. Z., *General Theory of Shells and Its Applications in Engineering*, NASA TT F-99, April 1964, pp. 521-532.

<sup>4</sup> Mushtari, K. M. and Galimov, K. Z., *Nonlinear Theory of Thin Elastic Shells*, NASA TT F-62, 1961, pp. 304-313.

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<sup>5</sup> Wang, C. T. and Sullivan, D. P., "Buckling of Sandwich Cylinders under Bending and Combined Bending and Axial Compression," *Journal of the Aeronautical Sciences*, Vol. 19, No. 7, July 1952, pp. 468-470.

<sup>6</sup> Bijlaard, P. P. and Gallagher, R. H., "Elastic Instability of a Cylindrical Shell under Arbitrary Circumferential Variation of Axial Stress," *Journal of the Aerospace Sciences*, Vol. 27, No. 11, Nov. 1960, pp. 854-858.

<sup>7</sup> Seide, P. and Weingarten, V. I., "On the Buckling of Circular Cylindrical Shells under Pure Bending," *Journal of Applied Mechanics*, March 1961, pp. 112-116.

<sup>8</sup> Johns, D. J., "On the Linear Buckling of Circular Cylindrical Shells under Asymmetric Axial Compressive Stress Distributions," *Journal of the Royal Aeronautical Society*, Vol. 70, Dec. 1966, pp. 1095-1097.

<sup>9</sup> Lakshmikantham, C. and Gerard, G., "On the Bending Elastic Stability of Isotropic Cylinders," *Journal of the Royal Aeronautical Society*, Vol. 71, Feb. 1967, pp. 136-138.

<sup>10</sup> Flügge, W., "Die Stabilität der Kreiszyinderschale," *Ingenieur-Archiv*, Vol. 3, 1932, pp. 463-506.

<sup>11</sup> Block, D. L., "Buckling of Eccentrically Stiffened Orthotropic Cylinders under Pure Bending," TN D-3351, March 1966, NASA.

<sup>12</sup> Lu, S. Y. and Nash, W. A., "Elastic Instability of Pressurized Cylindrical Shells under Compression or Bending," *Proceedings of the 4th U.S. National Congress of Applied Mechanics*, Univ. of California, Berkeley, Calif., June 1962, pp. 697-704.

<sup>13</sup> Yao, J. C., "Large-Deflection Analysis of Buckling of a Cylinder under Bending," *Journal of Applied Mechanics*, Dec. 1962, pp. 708-714.

<sup>14</sup> Timoshenko, S. and Gere, J. M., *Theory of Elastic Stability*, 2nd. ed., McGraw-Hill, New York, 1961, p. 484.

<sup>15</sup> Gerard, G., "Bending Tests of Thin-Walled Sandwich Cylinders," *Journal of the Aeronautical Sciences*, Vol. 20, No. 9, Sept. 1953, pp. 639-641.

<sup>16</sup> Gerard, G. and Becker, H., *Handbook of Structural Stability, Part III—Buckling of Curved Plates and Shells*, NACA TN 3783, 1957, pp. 31-38.

<sup>17</sup> Lundquist, E. E. and Burke, W. F., "Strength Tests of Thin-Walled Duralumin Cylinders of Elliptic Section," TN 527, 1935, NACA.

<sup>18</sup> Suer, H. S. et al., "The Bending Instability of Thin-Walled Unstiffened Cylinders Including the Effects of Internal Pressure," *Journal of the Aeronautical Sciences*, Vol. 25, No. 5, May 1958, pp. 281-287.

## Errata: "Buckling of Circular Cylindrical Shells with Multiple Orthotropic Layers and Eccentric Stiffeners"

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**I**N the above paper,

- 1) Equation (6) should read

$$\chi_3 = -2w_{,zy}$$

- 2) Equation (10) should read

$$\delta N_z = (B_{11} + E_s A_s/b)\epsilon_1 + \dots$$

- 3) Equation (11) should read

$$\delta M_y = C_{12}\epsilon_1 + (C_{22} + \bar{z}_r E_r A_r/a)\epsilon_2 + \dots$$

$$\delta M_{zy} = -C_{33}\epsilon_3 - \dots$$

- 4) Equation (13) should read

$$\delta N_x = B_{11}\epsilon_1 + B_{12}\epsilon_2 + C_{11}\chi_1 + C_{12}\chi_2$$

$$\delta N_y = B_{12}\epsilon_1 + B_{22}\epsilon_2 + C_{12}\chi_1 + C_{22}\chi_2$$

$$\delta N_{zy} = B_{33}\epsilon_3 + C_{33}\chi_3$$

- 5) Equation (15) should read

$$\delta N_{zy} = B_{zy}\epsilon_3$$

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